RESONANCE INTERACTION OF SPATIAL DISTURBANCES IN A BOUNDARY LAYER

M. B. Zel'man and I. I. Maslennikova UDC 532.517

A study of nonlinear disturbance growth mechanisms is an important problem in boundary-layer transition. The subharmonic region in transition, observed in [1], is characterized by rapid excitation of subharmonic driving frequencies, filling up of the low-frequency spectrum, and the formation of spatial distribution of even small amplitude disturbances. Certain features of the critical stages of such a process can be explained [2] by the amplification of Tollmien–Schlichting waves in symmetric Craik-type triads [3]. Their growth mechanism is wave resonance. The existing experimental [4-6] and theoretical [7-12] data confirm the relation between subharmonic transition and the evolution of synchronized disturbances. The existing models consider parametric instability of a pair of spatial waves in the given two-dimensional wave field [9, 10], bifurcation of self-excited triad states [8], and nonlinear evolution of resonant Tollmien-Schlichting waves [2, 3, 7, 11], but they are limited to the analysis of configurations that are symmetric in relation to the direction of the mean flow. The objective of this paper is to investigate the interaction of disturbances in nonsymmetric triads and their role in the formation of spatial structure and the spectrum of the initial stages of transition, and to interpret the experimental data.

The elementary model for such a study is an isolated triad of Tollmien–Schlichting waves. In the first nonlinear approximation of the weakly nonlinear theory, the disturbance field may be expressed through the stream function

$$\psi(x, y, t, z) = \sum_{j=1}^{m} A_j \varphi_j e^{i\theta_j(x,t,z)}, \qquad (1)$$

where $\theta_j = -\omega_j t + \int \alpha_j dx + \beta_j z;\ \varphi_j(y)$ and the dispersion relation $\omega_j + i\gamma_j = \Omega(\alpha_j, \beta_j)$ are determined by Orr–Sommerfeld equation with a locally parallel flow assumption, and the complex amplitude $A_j(x) = A_j^{\circ} e^{\nu_j t}$ satisfies the system (for m = 3) of nonlinear equations [2]. Assuming steady flow and transverse (along the z axis) homogeneity $(\partial/\partial t)A_j^{\circ} = (\partial/\partial x)A_j^{\circ} = 0$ such a system takes the form

$$\left(v_1 \frac{\partial}{\partial x} - \gamma_1\right) b_1 = S_1 b_2 b_3, \quad \left(v_{2,3} \frac{\partial}{\partial x} - \gamma_{2,3} - i\Delta\theta_{2,3}\right) b_{2,3} = S_{2,3} b_1 b_{3,2}^{*}, \qquad (2)$$

$$b_{j0} = b_j(x_0) = a_j(x_0) \exp(i\,\Phi_j(x_0)),$$

where $\Delta\theta_{2,3} = -(1/2)\ (\Delta\omega - v_{2,3}\Delta\alpha - w_{2,3}\Delta\beta)$; $\Delta(\omega, \alpha, \beta) = (\omega, \alpha, \beta)_1 - (\omega, \alpha, \beta)_2 - (\omega, \alpha, \beta)_3$; the following have been replaced: $b_1 = A_1$, $A_{2,3} = b_{2,3}\exp[(-i/2)(\Delta\omega t - \int\Delta\alpha dx - \Delta\beta z)]$.

Coefficients v_j, w_j, and S_j are determined by solving the homogeneous and nonhomogeneous Orr–Sommerfeld equations [2]. The present computations were carried out for disturbances in the Blasius boundary layer. Dimensional Z-components of wave vectors and frequency were left invariant; the Reynolds number Re was based on displacement thickness. We consider the behavior of the system (2) for different wave-vector configurations.

The relations $a_j(\mathrm{Re}) = |A_j|$ for quasisymmetric triads ($\omega_1 = 2\omega_2 = 2\omega_3$, $\omega_j = F_j\mathrm{Re}$, $F_1 = 115\cdot 10^{-6}$, $\beta_2 = -\beta_3 = 0.15$) with $\beta_1 = 0;\ 0.03;\ 0.06$ (the values of β_j correspond to Re at the initial section) are shown in Fig. 1. The case $\beta_1 = 0$ results in a symmetric triad. The equality of initial intensities of three-dimensional waves ($a_2(x_0) = a_3(x_0)$) ensures their matching throughout the growth region (curve 2). The behavior of a_1 is shown by curve 1. There is an explosive interaction of disturbances [2, 7]. An increase in β_1 changes the picture (lines 3-5 and 6-8 correspond to a_1, a_2, and a_3 for $\beta_1 = 0.03$ and 0.06, respectively). Growths decrease and do not agree for a_j. Local damping of the carrier disturbance (a_1) may exceed the value determined by linear theory (dotted line), which indicates the breakdown of the explosive nature of resonance. Nevertheless, rapid growth of low-frequency oscillations remains unchanged.

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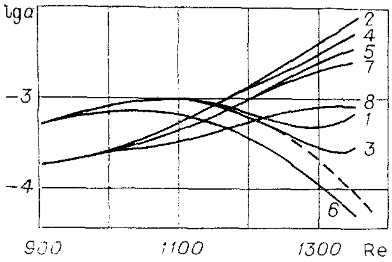

Fig. 1

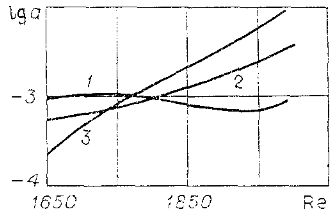

Fig. 2

The above-mentioned feature appears even in other types of triads. Computing data for wave interactions with $\omega_1 = 2\omega_2 = 2\omega_3$, $\beta_1 = \beta_2 + \beta_3$($F_1 = 59 \cdot 10^{-6}$, $\beta_1 = 0.15$, $\beta_2 = 0.2$, $\beta_3 = -0.05$) are shown in Fig. 2. The number of the curve corresponds to wave index j. Variation in initial phases $\Phi = \Phi_{10} - \Phi_{20} - \Phi_{30}$, as also in the case of a quasisymmetric triad, qualitatively maintains the variation in amplitudes. Triad configurations from two-dimensional a_1 ($F_1 = 105 \cdot 10^{-6}$, $\beta_1 = 0$) and a pair of three-dimensional waves a_2, a_3 ($F_2 = 38.10^{-6}$, $\beta_2 = 0.16$, $F_3 = 67 \cdot 10^{-6}$, $\beta_3 = -0.23$) (Fig. 3) are more sensitive to Φ. When $\Phi = 0$ (dashed lines), amplifications of a_2 and a_3 monotonically and structurally correspond to those mentioned earlier. A change in phase $\Phi = 3\pi/2$ (continuous curve) appreciably alters the trajectory. Amplitudes a_2 and a_3 oscillate, while a_2 could fall to the background noise level ($a \leqslant 10^{-7}$), and the rapid growth begins only after this. The complexity of such a motion and its sensitivity to change in the initial parameters are interesting from the point of view of randomization of the disturbance field.

It appears that the effects of resonant interaction are observed as long as there are transversely propagating (z axis) waves. Otherwise, the nonlinear effect in (2) is not negligible all the way up to $a_j \leqslant 10^{-1}$.

The results obtained indicate the importance of nonsymmetric triad resonance to excite low-frequency spatial disturbances. At the same time it is confirmed that maximum growth is attained in the symmetric structure, which explains its prominence under standard test conditions with vibrating ribbon [1, 5]. Here the source of the formation of such a structure is the interaction of the given plane wave with noise disturbances.

The corresponding model (two-dimensional wave with subharmonic spatially symmetric wave packets) was investigated in [7] without "cross" connections.

Consider the effects of such interactions in the basic system comprising five waves: $A_1(\omega_1, \alpha_1, \beta_1 = 0)$, $A_{2,3}(\omega_2 = \omega_3, \alpha_2 = \alpha_3, \beta_2 = -\beta_3)$, $A_{4,5}(\omega_4 = \omega_5, \alpha_4 = \alpha_5, \beta_4 = -\beta_5)$. The disturbance field is described by stream function (1) with m = 5 and the equations for amplitude take the form

$$\left(v_1 \frac{\partial}{\partial x} - \gamma_1\right) b_1 = S_1 b_2 b_3 + S_2 b_4 b_5 + E\,(b_4 b_3 + b_2 b_5),$$

$$\left(v_{2,3} \frac{\partial}{\partial x} - \gamma_{2,3}\right) b_{2,3} - \frac{1}{2}\,(\Delta\omega_1 - v_{2,3}\Delta\alpha_1 \mp w_{2,3}\Delta\beta)\, b_{2,3} = b_1 \left(C_1 b^*_{3,2} + D_1 b^*_{5,4}\right), \qquad (3)$$

$$\left(v_{4,5} \frac{\partial}{\partial x} - \gamma_{4,5}\right) b_{4,5} - (\Delta\omega_2 - v_{4,5}\Delta\alpha_2 \pm w_{4,5}\Delta\beta)\, b_{4,5} = b_1 \left(D_2 b^*_{3,2} + C_2 b^*_{5,4}\right),$$

$$b_j(x_0) = b_{j0},$$

$$b_1 = A_1, \quad b_{2,3} = A_{2,3} \exp i \left(\Delta\omega_1 t - \int \Delta\alpha_1 dx \mp \Delta\beta z\right),$$

$$b_{4,5} = A_{4,5} \exp i \left(\Delta\omega_2 t - \int \Delta\alpha dx \pm \Delta\beta z\right), \quad a_j = |b_j|.$$

The coupling in nonsymmetric triads is given by the terms (E, D_1, and D_2).

Results of numerical solutions of the system (3) with $\omega_1 = 2\omega_{2,3} = 2\omega_{4,5}$ ($F_1 = 115 \cdot 10^{-6}$, $\beta_2 = 0.136$, $\beta_4 = 0.156$ (Re = 900)) are shown in Fig. 4. It is seen that the interaction is of an explosive nature. Furthermore, the system is not only symmetricized, but thanks to nonsymmetric couplings the strengths of all its waves are equalized. It is sufficient to have one wave above the noise level (e.g., A_2) so that the interaction with a two-dimensional wave leads to the excitation of a whole packet from the noise. The above-described organization, which is a direct result of resonance, is absent in the case [7] that does not take them

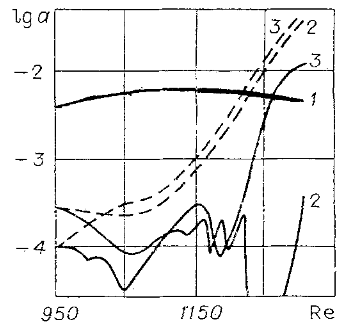

Fig. 3.

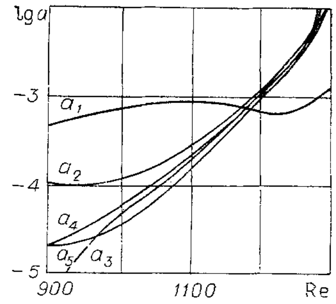

Fig. 4

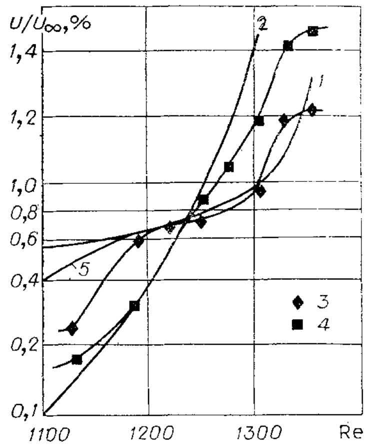

Fig. 5

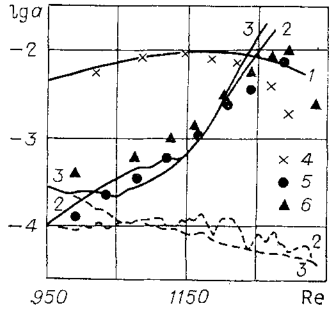

Fig. 6

into account. On the basis of strengths a_j the system behaves similarly to an isolated Craik triad but differs by a more complex spatial distribution of the disturbance field. As a result, it is possible to expect agreement of experimental and theoretical (for the triads) growths with the divergence of the values (α/β) of the ratio of wave-vector components.

A comparison of numerical computation of triad $a_2 = a_3$, $\omega_1 = 2\omega_2 = 2\omega_3$, $\beta_2 = -\beta_3$ ($F_1 = 106.10^{-6}$, $\beta_2 = 0.24$) with measurements [5] is shown in Fig. 5. There is a close agreement of resulsts; theoretical data (a_1 and a_2) are given by curves 1 and 2 and the corresponding experimental data by points 3 and 4. Quantitative disagreement with the initial segment is reduced by taking nonparallelness of the flow. The latter has little effect on the behavior of $a_{2,3}$, for which parametric amplification dominates but a_1 is appreciably deformed (with correction for nonparallelness, the curve 1 takes the position 5). Agreement in the segment ($Re \leqslant 1250$) is broken with increase in strengths. In this case, obviously, it is necessary to consider higher-order nonlinear effects which stabilize the growth of amplitudes. The energy transfer to the lower end of the spectrum may be a factor in the stabilization.

In the above-considered system, all waves were synchronized in symmetric triads. An example, when symmetrized pairs and plane waves are not resonant but there is a coupling is nonsymmetric configuration, is shown in Fig. 6 (in the system (3), the contribution of terms with $S_{1,2}$ and $C_{1,2}$ happens to be negligible). The behavior of a_1, $a_2 = a_3$, $a_4 = a_5$ ($F_1 = 105\cdot10^{-6}$, $F_2 = 38\cdot10^{-6}$, $F_3 = 67\cdot10^{-6}$, $\beta_2 = -\beta_3 = 0.16$, $\beta_4 = -\beta_5 = 0.25$ for $Re = 950$) is described by curves 1-3 (without cross interactions, the shape of $a_{2,3}$ and $a_{4,5}$ is shown by dashed lines).

The formation of such a system can explain the experimental results [12] in which disturbances $F_1 = 105\cdot10^{-6}$ and $F_0 = 143\cdot10^{-6}$ were introduced in the boundary layer on a flat plate with vibrating ribbon. It was found that spectral components $F_2 = F_0 - F_1$ and $F_3 = 2F_1 - F_0$ experience anomalous amplification and as a result exceed the strength of the given waves. Within the framework of the two-wave nonlinear interaction model such a result contradicts theoretical explanations.

An explanation for this fact may be given if it is possible to relate disturbances with coincident frequencies and the characteristic waves of the flow (there are no phase measurements in [12]). Then the disturbances with frequencies (F_1, F_2, and F_3) are capable of forming a resonant system whose numerical values are given in Fig. 6. The given disturbance helps increase initial amplitudes $a_{2,3}$ and $a_{4,5}$, whereas Tollmien–Schlichting waves generated by them do not participate in the interactions and rapidly damp out according to linear theory. The measured data agree well with computations in the region where the approximation is valid. Experimental results (a_1, $a_2 = a_3$, $a_4 = a_5$) are shown by points 4-6.

The data obtained make it possible to concude the important role of interactions in nonsymmetric triads during transition. Such interactions effectively influence the dominating spatially symmetric structure of the disturbance field and under certain conditions [12] completely determine it. The significant characteristic of the above mechanism is the appreciable energy transfer by low frequency disturbances as a result of which it is possible to effect a cascading process of exciting longer wavelength spatial fluctuations. On account of higher-order nonlinearity it is possible to carry out simultaneous saturation of the spectrum also in the broad band of high frequencies. Such an evolution is capable of randomization of the flow and, apparently, explains results of observations of the subharmonic transition process.

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